Applications of Fractional Calculus in the Stability and Convergence Analysis of Fractional Order Differential Equations

Synopsis of the Research Proposal

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1. Introduction:

(a) Necessity of the proposed research work: Fractional calculus (FC) is a branch of mathematics that extends the traditional concepts of differentiation and integration to non-integer and even complex orders. The origins of this field can be traced back to the historic correspondence between Leibniz and L'Hôpital. The idea underlying fractional calculus is as old as conventional calculus itself, which was independently developed by the British scientist Isaac Newton (1642–1727) and the German mathematician Gottfried Wilhelm Leibniz (1646–1716). Alongside his contributions to the development of calculus, Leibniz also introduced the symbolic representation of derivatives, expressing the derivative of y = I(τ) with respect to τ and used the symbol $\frac{d^k y}{d\tau^{\mathrm{k}}} = D^k y$ where k is a non-negative integer representing the order of the derivative.

The exchange of a letter between Leibniz and de L'H*ô*pital [33] provided evidencethat the subject of fractional calculus developed almost simultaneously with thebirth of classical calculus. In this letter, Leibniz raised following question, "Can themeaning of derivatives with integer order be generalized to derivatives with non-integer orders?". L'H*ô*pital was also curious about that question and then he askedLeibniz "What if the order will be 1/2?". The letter dated Sept. 30, 1695 wasthe actual origination of the fractional calculus. Leibniz replied: "It will lead to aparadox, from which one day useful consequences will be drawn." In these wordsfractional calculus was born. S.F. Lacroix (1819) [32] mentioned derivative of arbitrary order and also obtained the well-known half derivative.

In 1974, the University of New Haven (USA), hosted the first global conference onfractional calculus. The second and third international conferences were held inNihon University, Tokyo (Japan), and the University of Glasgow (Scotland), respectively. Fractional calculus was at its peak in the nineteenth century. This theorywas established in this century by Euler, Liouville, Riemann etc. The list of mathematicians who provided important contributions up to the middle of 20th Century, includes Atangana et al. [6, 7] Caputo et al. [8, 9, 10], Davis [11, 12], Duran et al. [17], Fourier [19], Galue et al. [20], Hardy and Littlewood [24], Kober [31], Letnikov [34, 35], Pandey and Srivastava [41], Riesz [44], Saigo and Raina [45], Saigo et al. [46], Srivastava and Saxena [47], Srivastava et al. [48], Zygmund[53], and several others.

Fractional calculus is a branch of mathematics that extends the traditional concepts of integral and differential calculus operators [26, 27], much like how fractional exponents generalize integer exponents. The theory of fractional calculus operators has been investigated for many years, with comprehensive historical developments presented in the works of Kilbas et al. [30], Miller and Ross [37], Nishimoto [39], Oldham and Spanier [40], and Podlubny [42], among others. Over the past three decades, the concept of fractional calculus encompassing derivatives and integrals of arbitrary real or complex order has gained significant attention due to its successful applications across diverse fields of science and engineering. These include, for instance, fluid mechanics, rheology, diffusion-like transport processes, electrical networks, control theory, viscoelasticity [36], electrochemical corrosion, and chemical physics.

A mathematical model is a representation of a system expressed through mathematical concepts and notation. The process of constructing such a representation is known as mathematical modeling. These models are widely utilized in the natural sciences, engineering, and social sciences to analyze, predict, and interpret complex phenomena.

Blum and Lei (2007) highlighted the importance of clarifying the fundamental concepts and terminology associated with mathematical models and the modeling process. In a similar vein, Lesh and Doerr (2003) described mathematical models as applied systems composed of elements, relationships, operations, and governing rules, all organized in a structured manner. These models serve to explain, predict, and control system behavior. Furthermore, Yoon, Dreyfus, and Thomas (2010) pointed out that the modeling process typically begins with a real-world problem, which is translated into mathematical form, solved, and then interpreted in relation to the original context.

Hence, real-world problems serve as the foundation for modeling activities, providing an effective means to illustrate the connection between mathematics and everyday experiences while enhancing conceptual understanding. In recent years, mathematical modeling incorporating fractional calculus has attracted significant interest for analyzing the dynamic behavior of complex biological and physical systems.

(b) Utility of the proposed research work:

A function which satisfies a differential equation is called solution of differential equation. Solutions of fractional order differential equations are the main reasons for the renewed interest in the area of fractional calculus. The solutions coming from fractional order differential equations appear to describe real-life data better, compared to the solutions of the corresponding integer order differential equations. The reason may be that integer order derivatives are local activities at a point whereas fractional order derivatives [38] are certain fractional integrals thereby describe global activity on an interval rather than a point, in one variable case. The real-life situation in the ideal case may be confined to a given point but in reality the situation may be around the point. Unlike ODEs, it is difficult to find analytical solutions for fractional order derivative. Moreover due to the complex geometries of most of the fractional differential equations [14] they lack exact solution. Hence numerical methods need to be developed to exact solutions. There are numerous schemes to gain the analytical and numerical solutions of fractional order differential equations such as the generalized Riccati equation mapping method [1], homotopy analysis transform method [50], variational iteration method [43], Collocation methods [2, 4, 13], Tikhonov regularization method [15, 16], reproducing kernel method [3, 5, 51], homotopy perturbation method [25], optimal homotopy analysis method [23], Adomain decomposition method [18], residue power series method [49], Laplace Adomain decomposition method [28, 52], homotopy perturbation transform method [21, 29], fractional reduced differential transform method [22] etc.

Fractional-order epidemic models provide a more accurate representation of disease dynamics by incorporating memory effects and hereditary properties, unlike classical integer-order models. These models have been widely applied to study the spread of infectious diseases such as Ebola Virus [56], HBV Model [57], Smoking Model [58], Glucose Model [59], COVID-19 [60], tuberculosis, and malaria [61]. The fractional derivatives allow a better understanding of long-term interactions between infected and susceptible populations, leading to more precise predictions of disease outbreaks and control strategies. Researchers like Diethelm and Magin [54, 55] have explored the advantages of fractional calculus in epidemiology, demonstrating its effectiveness in capturing complex biological processes.

(c) Importance of the proposed research work:

Analytical techniques for FDE`s provide a powerful and fundamental framework to the exploration of systematic procedures leading

- To identify the problem
- Develop the novel methods using generalized integral transforms
- Obtain solution using suitable method with generalized transforms
- Analysis the numerical result with conclusion
- Validation of result by comparing with existing results

In recent years, FDEs have been increasingly employed to describe complex phenomena in fluid mechanics, biology, electromagnetism, diffusion, and other physical processes. In light of this, the present research focuses on solving fractional differential equations to obtain approximate and numerical solutions using suitable analytical techniques. The results will demonstrate the efficiency and accuracy of the proposed methods.

Mathematical modeling involves establishing a mathematical model based on real-world problems, solving and calculating the mathematical model, and then applying the calculated results to solve real-life problems. The essence of a mathematical model is dynamic simulation, rather than a fixed way of thinking. It utilizes mathematical symbols, formulas, languages, graphics, etc., to abstract, summarize, and describe the essence of a problem, thereby explaining certain objective phenomena and developmental laws in life.

Mathematical models are not limited to specific fields but are also used in music, linguistics, and philosophy (for example, extensively in analytic philosophy). A model can aid in explaining a system, studying the effects of different components, and making predictions about behavior.

These applications are sufficient to motivate us to delve deeper into the study of mathematical modeling systems. It is now imperative to investigate fractional differential equations (FDEs) further for their analytical and numerical solutions using various hybrid methods. Therefore, we will continue our research work in the field of mathematical modeling, focusing on analytical techniques.

2. Literature Review:

1) Kumar, D., Singh, J., Kumar, S., [62].

In this paper, the authors employed the new Homotopy Perturbation Transform Method (HPTM) to solve the time-fractional Navier–Stokes equations in cylindrical coordinates and explore their applications. The obtained results were further analyzed and interpreted using graphical representations.

2) Shaikh, A., Tassaddiq, A., Nisar, K.S., Baleanu, D., [63].

In this paper, the authors introduced the Iterative Laplace Transform Method (ILTM) incorporating the Caputo–Fabrizio derivative operator to obtain approximate solutions of the FitzHugh–Nagumo (F–N) equation and the Fisher equation. They also established the conditions for existence and uniqueness of the solutions using the fixed-point theorem and the contraction principle, respectively.

3) Wang, L., Wu, Y., Ren, Y., Chen, X., [64].

In this paper, the authors solved the initial-value problem for time-fractional partial differential equations using two analytical approaches: the Residual Power Series Method (RPSM) and the Homotopy Analysis Transform Method (HATM). They have established that show that both schemes give the similar solution for $\hbar = -1$ and interpreted their results with the help of graphs.

4) Prakash, A., Prakasha, D.G., Veeresha, P., [65].

In this paper, the authors employed the q-Homotopy Analysis Transform Method (q-HATM) in the Caputo sense to obtain numerical solutions of the time-fractional Navier–Stokes equations for unsteady viscous fluid flow. The results were further analyzed and interpreted using graphical representations.

5) Wang, K., Liu, S., [66].

In this paper, the authors obtained approximate solutions of the nonlinear time-fractional Fornberg–Whitham equation using the New Iterative Transform Method (NITM) and a Modified Fractional Homotopy Analysis Transform Method (MFHATM). They also concluded that both methods yield similar solutions for $\hbar = -1$.

6) Suleman, M., Wu, Q., Abbas, G., [67].

In this paper, the authors obtained both series and exact solutions of the (2+1)-dimensional coupled Burgers' equations using the Elzaki Homotopy Analysis Method (EHPM). The results were further analyzed and interpreted through graphical representations.

7) Prakash, A., Kumar, M., Baleanu, D., [68].

In this paper, the authors applied the New Iterative Sumudu Transform Method (NISTM) to obtain numerical solutions of the nonlinear time-fractional Zakharov–Kuznetsov (FZK) equation. The results were further analyzed and interpreted using graphical representations.

8) Singh, B.K., Kumar, P., [69].

In this paper, the authors employed the Homotopy Perturbation Transform Method (HPTM) for the numerical solution of initial-value autonomous systems of time-fractional partial differential equations and generalized Burgers' equations with proportional delay. The numerical results and their corresponding graphs were analyzed to study the behavior of the solutions.

9) Mohamod, M.S., Al-Malki, F., Al-humyani, M., [70].

In this paper, the authors applied the Homotopy Analysis Transform Method (HATM) to obtain analytical solutions in the form of convergent series. The results were further analyzed and interpreted using graphical representations.

10) Veeresha, P., Prakasha, D.G., Baskonus, H.M., [71].

In this paper, the authors obtained an approximate analytical solution of the fractional smoking

epidemic model using the q-Homotopy Analysis Transform Method (q-HATM). They also demonstrated that the considered nonlinear mathematical model can effectively describe the evolution of smoking within a population and its impact on public health in a community.

11) Saravanan, A., Magesh, N., Christopher, J., [72].

In this paper, the authors obtained solutions of the first-order nonlinear hyperbolic partial differential equation using the Reduced Differential Transform Method (RDTM) and the Iterative Laplace Transform Method (ILTM). They also demonstrated the simplicity, accuracy, and reliability of RDTM compared to ILTM.

3. Present Study:

a. To study the fundamental concepts and properties of fractional calculus relevant to fractional order differential equations.

b. To explore hybrid analytical and numerical techniques for solving fractional differential equations, with a focus on stability and convergence analysis.

c. To investigate different types of fractional derivatives and their impact on the behaviour of fractional differential equations.

d. To examine the efficiency and accuracy of the proposed methods in comparison with existing methods.

4. Methodology:

a. **Procedure:**

- To identify the problem
- Solution using suitable method
- Analysis the numerical result with conclusion
- Validation of result

b. **Methods:**We will use following methods in above proposed work,

- Homotopy analysis Shehu transform method (HASTM),
- Laplace Residual power series method (LRPSM),
- Iterative Shehu transform method (ISTM),
- q-Homotopy analysis Ezaki transform method (q-HAETM).

c. **Software's:**

- MATLAB
- SCI-LAB
- LaTeX

5. Data Analysis/Discussion:

Fractional differential equations (FDEs) have gained considerable importance and popularity in recent years, primarily because of their proven applicability across various domains of science and engineering. They are increasingly utilized to model a wide range of complex phenomena, including:

i. fluid flow,

ii. diffusion processes,

iii. oscillatory systems,

iv. electrical networks,

v. chemical physics,

vi. dynamical processes in self-similar media,

vii. transport phenomena in porous structures,

and numerous other physical processes.

Expected outcomes:

The solutions derived from fractional-order differential equations have shown superior capability in capturing real-life data patterns compared to those obtained from integer-order differential equations. This superiority arises from the inherent nature of fractional derivatives, which are integral operators that account for the entire history or global behavior over an interval, rather than local behavior at a single point, as is the case with integer-order derivatives. Consequently, fractional-order models are better suited for describing systems with memory and hereditary effects. The proposed method yields an analytical series solution for fractional-order differential equations, with results demonstrating fast convergence towards the exact solution.

6. Limitations & Delimitations

1. **High Computational Complexity** – Solving FDEs requires more time and resources than classical methods.
2. **No Universal Solution Method** – A single approach cannot solve all types of fractional differential equations.
3. **Parameter Sensitivity** – Small changes in fractional order (α) or initial conditions can significantly affect results.
4. **Limited Experimental Data** – Few real-world datasets are available to validate models.
5. **Requires Specialized Expertise** – Needs strong mathematical background for proper implementation.
6. **Software Constraints** – Limited built-in support for fractional calculus in common tools.
7. **Simplifying Assumptions** – Some models may not fully capture complex physical phenomena.

7. Work Plan and Expected duration of the investigation:

	Course Name\Month	3	6	9	12	15	18	21	24	27	30	33	36
1.	Course work	✔	✔										
2.	Literature Review	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
3.	Work on Specific Research problems			✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
4.	Paper Writing				✔	✔	✔	✔	✔	✔	✔	✔	✔
5.	Communication of Papers				✔	✔	✔	✔	✔	✔	✔	✔	✔
6.	Thesis Writing											✔	✔
7.	Submission												✔

Bibliography:

1. Akinyemi, L., Şenol, M. and Osman, M.S., 2022. Analytical and approximate solutions of nonlinear Schrödinger equation with higher dimension in the anomalous dispersion regime. Journal of Ocean Engineering and Science, 7(2), pp.143-154.
2. Ali, K.K., Abd El Salam, M.A., Mohamed, E.M., Samet, B., Kumar, S. and Osman, M.S., 2020. Numerical solution for generalized nonlinear fractional integro-differential equations with linear functional arguments using Chebyshev series. Advances in Difference Equations, 2020, pp.1-23.
3. Al-Smadi, M., Abu Arqub, O. and El-Ajou, A., 2014. A Numerical Iterative Method for Solving Systems of First-Order Periodic Boundary Value Problems. Journal of Applied Mathematics, 2014(1), p.135465.
4. Amin, R., Shah, K., Ahmad, H., Ganie, A.H., Abdel-Aty, A.H. and Botmart, T., 2022. Haar wavelet method for solution of variable order linear fractional integro-differential equations.
5. Arqub, O.A., Osman, M.S., Abdel-Aty, A.H., Mohamed, A.B.A. and Momani, S., 2020. A numerical algorithm for the solutions of ABC singular Lane–Emden type models arising in astrophysics using reproducing kernel discretization method. Mathematics, 8(6), p.923.
6. Atangana, A. and Baleanu, D., 2016. New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model. arXiv preprint arXiv:1602.03408.
7. Atangana, A. and Gómez-Aguilar, J.F., 2018. Decolonisation of fractional calculus rules: breaking commutativity and associativity to capture more natural phenomena. The European Physical Journal Plus, 133, pp.1-22.
8. Caputo, M. and Mainardi, F., 1971. A new dissipation model based on memory mechanism. Pure and applied Geophysics, 91, pp.134-147.
9. Caputo, M. and Fabrizio, M., 2015. A new definition of fractional derivative without singular kernel. Progress in Fractional Differentiation and Applications, 1(2), pp.1–13.
10. Caputo, M. and Fabrizio, M., 2016. Applications of new time and spatial fractional derivatives with exponential kernels. Progress in Fractional Differentiation and Applications, 2(1), pp.1–11.
11. Davis, H.T., 1924. Fractional operations as applied to a class of Volterra integral equations. American Journal of Mathematics, 46(2), pp.95–109.
12. Davis, H.T., 1927. The applications of fractional operators to functional equations. American Journal of Mathematics, 49(1), pp.123–142.
13. Dhawan, S., Machado, J.A.T., Brzeziński, D.W. and Osman, M.S., 2021. A Chebyshev wavelet collocation method for some types of differential problems. Symmetry, 13(4), p.536.
14. Diethelm, K. and Ford, N.J., 2002. Analysis of fractional differential equations. Journal of Mathematical Analysis and Applications, 265(2), pp.229–248.
15. Djennadi, S., Shawagfeh, N., Inc, M., Osman, M.S., Gómez-Aguilar, J.F. and Arqub, O.A., 2021. The Tikhonov regularization method for the inverse source problem of time fractional heat equation in the view of ABC-fractional technique. PhysicaScripta, 96(9), p.094006.
16. Djennadi, S., Shawagfeh, N. and Arqub, O.A., 2021. A fractional Tikhonov regularization method for an inverse backward and source problems in the time-space fractional diffusion equations. Chaos, Solitons & Fractals, 150, p.111127.

17. Duran, J.A., Kalla, S.L. and Srivastava, H.M., 1995. Fractional calculus and the sum of certain families of infinite series. Journal of Mathematical Analysis and Applications, 190(3), pp.738–754.
18. El-Sayed, M. and Kaya, D., 2005. Exact and numerical travelling wave solutions of Whitham–Broer–Kaup equations. Applied Mathematics and Computation, 167(2), pp.1339–1349.
19. Fourier, J., 1955. The Analytical Theory of Heat. New York: Dover Publications.
20. Galue, L., Kiryakova, V.S. and Kalla, S.L., 1993. Solution of dual integral equations by fractional calculus. Mathematica Balkanica, 7, pp.53–72.
21. Gondal, M.A. and Khan, M., 2010. Homotopy perturbation method for nonlinear exponential boundary layer equation using Laplace transformation, He's polynomials and Padé technology. International Journal of Nonlinear Sciences and Numerical Simulation, 11(12), pp.1145–1153.
22. Gupta, P.K., 2011. Approximate analytical solutions of fractional Benney–Lin equation by reduced differential transform method and the homotopy perturbation method. Computers & Mathematics with Applications, 61(9), pp.2829–2842.
23. Haq, S. and Ishaq, M., 2014. Solution of coupled Whitham–Broer–Kaup equations using optimal homotopy asymptotic method. Ocean Engineering, 84, pp.81–88.
24. Hardy, G.H. and Littlewood, J.E., 1932. Some properties of fractional integrals. MathematischeZeitschrift, 34, pp.409–439.
25. He, J., 1999. Homotopy perturbation technique. Computer Methods in Applied Mechanics and Engineering, 178(3), pp.257–262.
26. Katugampola, U.N., 2011. New approach to a generalized fractional integral. Applied Mathematics and Computation, 218, pp.860–865.
27. Katugampola, U.N., 2014. A new approach to generalized fractional derivatives. Bulletin of Mathematical Analysis and Applications, 6(4), pp.1–15.
28. Khan, M. and Hussain, M., 2011. Application of Laplace decomposition method on semi-infinite domain. Numerical Algorithms, 56(2), pp.211–218.
29. Khan, Y. and Wu, Q., 2011. Homotopy perturbation transform method for nonlinear equations using He's polynomials. Computers & Mathematics with Applications, 61(8), pp.1963–1967.
30. Kilbas, A.A., Srivastava, H.M. and Trujillo, J.J., 2006. Theory and Applications of Fractional Differential Equations. Amsterdam: Elsevier North-Holland Science Publishers.
31. Kober, H., 1940. On fractional integrals and derivatives. The Quarterly Journal of Mathematics, 11(1), pp.193–211.
32. Lacroix, S.F., 1819. Traité du calculdifférentiel et du calculintégral, 2nd ed. Courcier, Paris, 3, pp.409–410.
33. Leibniz, G.W., 1849. Letter from Hanover, Germany, September 30, 1695 to G.A. L'Hospital. In LeibnizenMathematischeSchrifien, Olms Verlag, Hildesheim, Germany, 2, pp.301–302.
34. Letnikov, A., 1868. Theory of differentiation with an arbitrary indicator (In Russian). MatemSbornik, 3, pp.1–68.
35. Letnikov, A., 1872. On explanation of the main propositions of differentiation theory with an arbitrary index (In Russian). Sbornik Mathematics, 6, pp.413–445.
36. Matlob, M. and Jamalil, Y., 2019. The concepts and applications of fractional order differential calculus in modelling of viscoelastic systems: A primer. Critical Reviews in Biomedical Engineering, 47(4), pp.249–276.
37. Miller, S.S. and Ross, B., 1993. An introduction to the fractional calculus and fractional differential equations. New York, Chichester, Brisbane, Toronto and Singapore: John Wiley and Sons.
38. Nieto, J.J. and Losada, J., 2015. Properties of a new fractional derivative without singular kernel. Progress in Fractional Differentiation and Applications, 1(2), pp.87–92.

39. Nishimoto, K., 1984. Fractional calculus. Koriyama, Japan: Descartes Press.
40. Oldham, K.B. and Spanier, J., 1974. The fractional calculus. New York, London: Academic Press.
41. Pandey, R.N. and Srivastava, H.M., 1993. Fractional calculus and its applications involving certain class of functional relations. Studies in Applied Mathematics, 89(2), pp.153–165.
42. Podlubny, I., 1999. Fractional differential equations. Mathematics in Science and Engineering, San Diego: Academic Press.
43. Rafel, M. and Daniali, H., 2007. Application of the variational iteration method to the Whitham-Broer-Kaup equations. Computers & Mathematics with Applications, 54, pp.1079–1085.
44. Riesz, M., 1949. L'intégrale de Riemann-Liouville et le problème de Cauchy. Acta Mathematica, 81, pp.1–222.
45. Saigo, M. and Raina, R.K., 1988. Fractional calculus operators associated with a general class of polynomials. Fukuoka University Science Reports, 18, pp.15–22.
46. Saigo, M., Raina, R.K. and Kilbas, A.A., 1993. On generalized fractional calculus operators and their compositions with axisymmetric differential operators of the potential theory on spaces f_(ρ,μ) and ⟦f'⟧_(ρ,μ). Fukuoka University Science Reports, 23, pp.133–154.
47. Srivastava, H.M. and Saxena, R.K., 2001. Operators of fractional integration and their applications. Applied Mathematics and Computation, 118(1), pp.1–52.
48. Srivastava, H.M., Saigo, M. and Owa, S., 1988. A class of distortion theorems involving certain operators of fractional calculus. Journal of Mathematical Analysis and Applications, 131(2), pp.412–420.
49. Wang, Y., Zhang, Y.F., Jiang, Z. and Iqbal, M., 2017. A fractional Whitham-Broer-Kaup equation and its possible application to tsunami prevention. Thermal Science, 21(4), pp.1847–1855.
50. Wang, L., Wu, Y., Ren, Y. and Chen, X., 2019. Two analytical methods for fractional partial differential equations with proportional delay. IAENG International Journal of Applied Mathematics, 49(1).
51. Yildirima, E.N., Akgul, A. and Inc, M., 2021. Reproducing kernel method for the solutions of non-linear partial differential equations. Arab Journal of Basic and Applied Sciences, 28(1), pp.80–86.
52. Yusufo-lu, E., 2006. Numerical solution of duffing equation by the Laplace decomposition algorithm. Applied Mathematics and Computation, 177(2), pp.572–580.
53. Zygmund, A., 1945. A theorem on fractional derivatives. Duke Mathematical Journal, 12(3), pp.455–464.
54. Diethelm, K. and Ford, N.J., 2010. The analysis of fractional differential equations. Lecture Notes in Mathematics, vol. 2004.
55. Magin, R.L., 2004. Fractional Calculus in Bioengineering. Critical Reviews in Biomedical Engineering, 32(1), pp.1–104.
56. Jacob, S.T., Crozier, I., Fischer, W.A., Hewlett, A., Kraft, C.S., Vega, M.A.D.L., Soka, M.J., Wahl, V., Griffiths, A., Bollinger, L. and Kuhn, J.H., 2020. Ebola virus disease. Nature reviews Disease primers, 6(1), p.13.
57. Farhan, M., Aljuaydi, F., Shah, Z., Alzahrani, E., Bonyah, E. and Islam, S., 2024. A fractional modeling approach to a new hepatitis B model in light of asymptomatic carriers, vaccination and treatment. Scientific African, 24, p.e02127.
58. Chauhan, R.P., Kumar, S. and Thota, S., 2025. Analyzing Tobacco Smoking Dynamics: Fractional and Fractal-Fractional Approaches. WSEAS Transactions on Systems, 24, pp.20-35.
59. Saber, S., Solouma, E., Alharb, R.A. and Alalyani, A., 2025. Chaos in fractional-order glucose–insulin models with variable derivatives: Insights from the Laplace–adomian decomposition method and generalized Euler techniques. Fractal and Fractional, 9(3), p.149.

60. Khan, H., Ahmad, F., Tunç, O. and Idrees, M., 2022. On fractal-fractional COVID-19 mathematical model. Chaos, Solitons & Fractals, 157, p.111937.
61. Rezapour, S., Etemad, S., Asamoah, J.K.K., Ahmad, H. and Nonlaopon, K., 2023. A mathematical approach for studying the fractal-fractional hybrid Mittag-Leffler model of malaria under some control factors. AIMS Mathematics, 8(2), pp.3120–3162.
62. Kumar, D., Singh, J., Kumar, S.: A fractional model of Navier-Stokes equation arising in unsteady flow of a viscous fluid. J. of the Associ. Of Arab Uni. for Basic and Appl. Sci. 17, 14-19, (2015).
63. Shaikh, A., Tassaddiq, A., Nisar, K.S., Baleanu, D.: Analysis of differential equations involving Caputo-Fabrizio fractional operator and ita applications to reaction-diffusion equations. Adv. In Diff. Eqs. 178, 1-14, (2019).
64. Wang, L., Wu, Y., Ren, Y., Chen, X.: Two analytical methods for fractional partial differential equations with proportional delay. IAENG Int. J. of Appl. Matn., 49 (1), (2019).
65. Prakash, A., Prakasha, D.G., Veeresha, P.: A reliable algorithm for time fractional Navier-Stokes equations via Laplace transform. Nonlinear Eng. 8, 695-701, (2019).
66. Wang, K., Liu, S.: Application of new iterative transform method and modified fractional homotopy analysis transform method for fractional Fornberg-Whitham equation, J. of nonlinear Sci. and Appl. 9, 2419-2433(2016).
67. Suleman, M., Wu, Q., Abbas, G.: Approximate analytic solution (2+1) dimensional coupled differential Burger's equation using Elzakihomotopy perturbation method, Alexandria Eng. J. 55, 1817-1826 (2016).
68. Prakash, A., Kumar, M., Baleanu, D.: A new iterative technique for a fractional model of nonlinear Zakharov-Kuznetsov equations via Sumudu transform, Appl. Math. and Comput. 334, 30-40 (2018).
69. Singh, B.K., Kumar, P.: Homotopy perturbation transform method for solving fractional partial differential equations with proportional delay, SeMA J. 75(1), 111-125 (2018).
70. Mohamod, M.S., Al-Malki, F., Al-humyani, M.: Homotopy analysis transform method for time-space fractional gas dynamics equation, Gen. Math. Notes 24(1), 1-16 (2014).
71. Veeresha, P., Prakasha, D.G., Baskonus, H.M.: Solving smoking epidemic model of fractional order using a modified homotopy analysis transform method, Mathematical Sci. 13, 115-128 (2019).
72. Saravanan, A., Magesh, N., Christopher, J.: A new computational method for smooth solution of first order nonlinear Cauchy problem, Int. J. of advances in Math. 2018(5), 14-24 (2018).